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IKKINCHI TARTIBLI GRONUOLL CHEGARALANISHLI BOSHQARUVLAR UCHUN TUTISH MASALASI

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Annotatsiya. Ushbu ma'ruzada boshqaruvlar Granoull chegaralanishga ega holda ikkinchi tartibli differensial o'yinlar uchun tutish masalasi o'rganiladi. Bunda quvlovchi uchun parallel quvish strategiyasi quriladi va uning yordamida tutish masalasi uchun yetarli shartlar keltiriladi.

Kalit so`zlar: Differensial o'yin, geometrik chegaralanish, parallel quvish strategiyasi, quvlovchi, qochuvchi, tezlanish, Granoull chegaralanishli.

THE SECOND ORDER GRONUOLL IS A CATCH ISSUE FOR BOUNDED CONTROLS.

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Annotation. This report explores the problem of retaining control in second-order differential games with Granule constraints. In this case, a parallel pursuit strategy is built for the pursuer, and with its help sufficient conditions are set for the task of capture.

Keywords: Differential game, geometric constraint, parallel chase strategy, pursuer, escape, acceleration, Granule constraint**.**

R^n fazoda **P** va **E** obyektlar berilgan va ularning harakatlari quyidagi differensial tenglamalarga asoslangan

P: $\ddot{x} = u$, $\dot{x}(0) - kx(0) = 0$, $\quad |u(t)|^2 \le \rho^2 + 2l\int_0^t |u(s)|^2 ds$, (1)

E: $\ddot{y} = v, \dot{y}(0) - ky(0) = 0$, $\quad |v(t)|^2 \le \sigma^2 + 2l\int_0^t |v(s)|^2 ds$, (2)

bu yerda x – **P** obyektning $\boldsymbol{R}^n$ fazodagi holati, $x_0 = x(0)$, $x_1 = \dot{x}(0)$–uning mos ravishda $t = 0$ vaqtdagi boshlang‘ich holati va boshlang‘ich tezligi; u– quvlovchining boshqariladigan tezlanishi bo‘lib $u:[0,\infty) \to \boldsymbol{R}^n$ va u vaqt bo‘yicha o‘lchanuvchi funksiya sifatida tanlanadi; barcha $|u(t)|^2 \le \rho^2 + 2l\int_0^t |u(s)|^2 ds$ shartni qanoatlantiruvchi bunday $u(\cdot)$ o‘lchanuvchi funksiyalar to’plamini $\boldsymbol{G}_p$ bilan belgilaymiz. y–**E** obyektning $\boldsymbol{R}^n$ fazodagi holati, $y_0 = y(0)$, $y_1 = \dot{y}(0)$ – uning mos ravishda barcha $|v(t)|^2 \le \sigma^2 + 2l\int_0^t |v(s)|^2 ds$ shartni qanoatlantiruvchi bunday $v(\cdot)$ o‘lchanuvchi funksiyalar to‘plamini $\boldsymbol{G}_E$ bilan belgilaymiz.

Ta’rif 1. Agar $(x_0,\ x_1,\ u(\cdot))$, $u(\cdot) \in \boldsymbol{G}_p$ uchlik berilgan bo’lsa, (1) tenglamaning quyidagi yechimiga quvlovchining harakat trayektoriyasi deyiladi

$$x(t) = x_0 + tx_1 + \int_0^t\int_0^s u(\tau)\,d\tau ds.$$

Ta’rif 2. Agar $(y_0,\ y_1, v(\cdot))$, $v(\cdot) \in \boldsymbol{G}_E$ uchlik berilgan bo‘lsa (2) tenglamaning quyidagi yechimiga qochuvchining harakat trayektoriyasi deyiladi

$$y(t) = y_0 + ty_1 + \int_0^t\int_0^s v(\tau)\,d\tau ds.$$

Ta’rif 3. (1)-(2) masala uchun tutish masalasi ([1]-[2]) yechilgan deyiladi, agar qochuvchining ixtiyoriy $v(\cdot)\in \boldsymbol{G}_E$ boshqaruv funksiyasi uchun quvlovchining shunday $u^*(\cdot)\in \boldsymbol{G}_p$ boshqaruv funksiya mavjud bo‘lsaki, biror chekli t^* vaqtda quyidagi tenglik bajarilsin

$$x(t^*)=y(t^*). \qquad (3)$$

Ta’rif 4. (1) – (2) masala uchun quvlovchining П-strategiyasi ([3]-[4]) deb quyidagi funksiyaga aytamiz,

$$u(v)=v-\lambda(v)\xi_0, \qquad (4)$$

bunda $\xi_0=\dfrac{z_0}{|z_0|}$, $\lambda(v)=(v,\xi_0)+\sqrt{(v,\xi_0)^2+\delta e^{2lt}}$, $\delta=\rho^2-\sigma^2\geq 0$, (v,ξ_0) – v va ξ_0 vektorlarning $\boldsymbol{R}^n$ fazodagi skalyar ko’paytmasi.

Teorema. Agar Granoull chegaralanishli ikkinchi tartibli differensial o‘yin (1)-(2) uchun quyidagi shart $\rho>\sigma$ o‘rinli bo‘lsa, u holda П-strategiya (4) yordamida tutish masalasi (0,t) yechiladi va obyektlar orasidagi yaqinlashish funksiyasi quyidagicha bo‘ladi

$$f(l,t,|z_0|,\rho,\sigma,k)=|z_0|(kt+1)-\frac{\rho-\sigma}{l^2}e^{lt}+\frac{\rho-\sigma}{l^2}+\frac{\rho-\sigma}{l}t$$

Isboti. Faraz qilamiz, agar qochuvchi ixtiyoriy $v(\cdot)\in \boldsymbol{G}_E$ bo‘lganda, quvlovchi esa (4) ko‘rinishdagi strategiyani tanlasin, u holda (1) va (2) tenglamalarga asosan quyidagi Karateodori tenglamasini topamiz

$$\ddot{z}=-\lambda(v(t))\xi_0, \qquad \dot{z}(0)-kz(0)=0,$$

Bundan boshlang’ich shartlarni berilishi bo’yicha quyidagi yechim aniqlanadi

$$z(t) = z_0(kt+1) - \xi_0 \int_0^t \int_0^s \lambda(v(\tau), \xi_0)\, d\tau ds$$

yoki

$$|z(t)| = |z_0|(kt+1) - \int_0^t \int_0^s ((v, \xi_0) + \sqrt{(v, \xi_0)^2 + \delta e^{2lt}})\, d\tau ds.$$

Lemmaga ko’ra quyidagi tengsizliklarni hosil qilamiz

$$|z(t)| \le |z_0|(kt+1) - \int_0^t \int_0^s e^{l\tau}(\rho - \sigma)\, d\tau ds \Rightarrow$$

$$|z(t)| \le |z_0|(kt+1) - \frac{\rho-\sigma}{l^2}e^{lt} + \frac{\rho-\sigma}{l^2} + \frac{\rho-\sigma}{l}t$$

Agar $f(l, t, |z_0|, \rho, \sigma, k) = |z_0|(kt+1) - \frac{\rho-\sigma}{l^2}e^{lt} + \frac{\rho-\sigma}{l^2} + \frac{\rho-\sigma}{l}t$ desak bu funksiyani nolga aylantiruvchi musbat t^* vaqtni topamiz.

$$\frac{\rho-\sigma}{l^2}e^{lt} = |z_0|(kt+1) + \frac{\rho-\sigma}{l^2} + \frac{\rho-\sigma}{l}t,$$

oxirgi tenglikni soddalashtirish orqali quyidagi tenglikni hosil qilamiz,

$$e^{lt} = t\left(\frac{|z_0|kl^2}{\rho-\sigma} + l\right) + \frac{|z_0|l^2}{\rho-\sigma} + 1$$

bunda $A = \frac{|z_0|kl^2}{\rho-\sigma} + l$, $B = \frac{|z_0|l^2}{\rho-\sigma} + 1$ bo‘lib, bu yerda $\rho > \sigma$, $B > 1$. Natijada quyidagi tenglikka ega bo‘lamiz

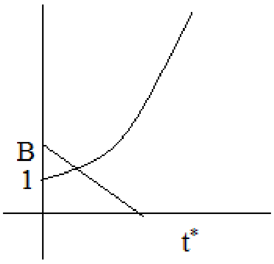
$$e^{lt} = At + B \qquad (5)$$

Tutish vaqtini aniqlash uchun (5) tenglamani quyidagi hollarini ko‘rib chiqamiz.

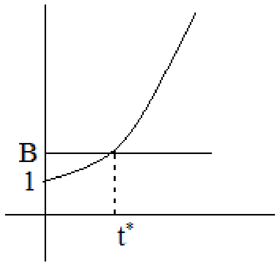
1. $A < 0 \Rightarrow k < \frac{\sigma-\rho}{|z_0|l}$ bo‘lsin. U holda (5) tenglama yagona $t^* > 0$ musbat yechim mavjud va bu yechim tutish vaqti bo‘ladi. (1-chizma)

2. $A = 0 \quad \Rightarrow \quad k = \dfrac{\sigma - \rho}{|z_0| l}$ bo‘lsin. U holda (5) tenglama yechimi $t^* = \dfrac{\ln\left(\dfrac{|z_0| l^2}{\rho - \sigma} + 1\right)}{l}$ bo‘lib, tutish vaqtini beradi.

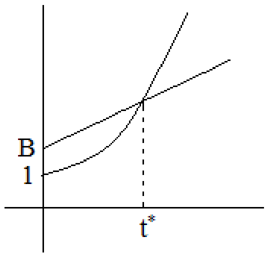
3. $A > 0 \Rightarrow k > \dfrac{\sigma - \rho}{|z_0| l}$ bo‘lsin. U holda (5) tenglama $t^* > 0$ musbat yechimi mavjud va bu yechim tutish vaqti bo‘ladi.



(1-chizma)



(2-chizma)



(3-chizma)

Foydalanilgan adabiyotlar

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