THE STRATEGY OF COORDINATED PURSUIT FOR A FIRST-ORDER DIFFERENTIAL GAME WITH GRONWALL-BELLMAN CONSTRAINTS

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Abstract. *In this work, we will consider a simple pursuit differential game of the fitst order when Gronwall-Bellman constraints imposed on control functions of the players. The proposed method substantiates the parallel approach strategy in this line differential game of the fitst order. The new sufficient solvability conditions are obtained for problem of the pursuit.*

Keywords: *Differential game, geometrical constraint, Gronwall-Bellman constraint, evader, pursuer, strategy, parallel pursuit.*

СТРАТЕГИЯ СОГЛАСНОГО ПРЕСЛЕДОВАНИЯ ДЛЯ ДИФФЕРЕНЦИАЛЬНОЙ ИГРЫ ПЕРВОГО ПОРЯДКА С ОГРАНИЧЕНИЯМИ ГРОНУОЛЛА-БЕЛЛМАНА

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Аннотация. *В работе рассматривается дифференциальная игра первого порядка при ограничениях Гронуолла-Беллмана на управления игроков. При этом предлагается стратегия параллельного преследования для преследователя и при помощи этой стратегии решается задача преследования.*

Ключевые слова: *дифференциальная игра, геометрическое ограничение, стратегия параллельного преследования, преследователь, ограничение Гронуолла-Беллмана, убегающий, скорость.*

Let in the space $\mathbf{R}^n$ the controlled object $\mathbf{P}$ (the pursuer), chases another object $\mathbf{E}$ (the evader). Suppose x and y are the locations of the pursuer and the evader respectively, and x_0, y_0 $(x_0 \neq y_0)$ are their initial locations. The motions of the objects are described by the equations

$$\mathbf{P}: \ \dot{x} = u, \ x(0) = x_0, \qquad (1)$$

$$\mathbf{E}: \ \dot{y} = v, \ y(0) = y_0, \qquad (2)$$

where $x, y, u, v \in \mathbf{R}^n, \ n \geq 2.$

In the present, the concept of the first type of Gronwall-Bellman constraint [1] for the control $u(\cdot)$ is introduced in the form

$$|u(t)|^2 \le \rho^2 + 2\int_0^t l(s)|u(s)|^2\, ds, \tag{3}$$

where ρ is a non-negative number and $l(s)$ is a non-negative function. The first type of Gronwall-Bellman constraint generalizes geometrical constraint when $l(s)=0$.

Similarly, the concept of the first type of Gronwall-Bellman constraint for the control $v(\cdot)$ is introduced in the form

$$|v(t)|^2 \le \sigma^2 + 2\int_0^t l(s)|v(s)|^2\, ds, \tag{4}$$

where σ is a non-negative number and $l(s)$ is a non-negative function.

Here, u is the velocity vector of the pursuer and here the temporal variation of u must be a measurable function $u(\cdot):[0,+\infty)\to \mathbf{R}^n$. We denote by $\mathbf{U}_{GB}$ the set of all measurable functions $u(\cdot)$ satisfying Gronwall-Bellman constraint (3) (briefly, GB-constraint).

Similarly, v is the velocity vector of the evader and here the temporal variation of v must be a measurable function $v(\cdot):[0,+\infty)\to \mathbf{R}^n$. We denote by $\mathbf{V}_{GB}$ the set of all measurable functions $v(\cdot)$ satisfying the GB-constraint (4).

Definition 1. For a pair of $(x_0, u(\cdot))$, $u(\cdot)\in \mathbf{U}_{GB}$ the solution of the equation (1), that is,

$$x(t) = x_0 + \int_0^t u(s)\,ds$$

is called a trajectory of the pursuer on interval $t \ge 0$.

Definition 2. For a pair of $(y_0, v(\cdot))$, $v(\cdot)\in \mathbf{V}_{GB}$ the solution of the equation (2), that is,

$$y(t) = y_0 + \int_0^t v(s)\,ds$$

is called a trajectory of the evader on interval $t \geq 0$.

Definition 3. The pair of classes of admissible controls introduced $(\mathbf{U}_{GB}, \mathbf{V}_{GB})$ defines differential game (1)-(4) with constraints of the Gronwall-Bellman type or briefly, GB-game.

Definition 4. In the GB-game, the pursuit problem is called to be solved if there exists such control function $u^*(\cdot) \in \mathbf{U}_{GB}$ of the pursuer for any control function $v(\cdot) \in \mathbf{V}_{GB}$ of the evader and the following equality holds at some finite time t^*

$$x(t^*) = y(t^*). \tag{5}$$

Definition 5. If $\rho \geq \sigma$, then

$$u_{GB}(t,v) = v - \lambda_{GB}(t,v)\xi_0 \tag{6}$$

is called Π_{GB}-strategy of the pursuer ([3]-[4]) in the GB-game, where $\lambda_{GB}(v,t) = (v,\xi_0) + \sqrt{(v,\xi_0)^2 + \rho^2 e^{2\int_0^t l(s)ds} - |v|^2}$, $\xi_0 = \dfrac{z_0}{|z_0|}$ and $\langle v, \xi_0 \rangle$ is the scalar product of the vectors v and ξ_0 in the space $\mathbf{R}^n$.

Note that

$$|u_{GB}(t,v)|^2 = |v|^2 + (\rho^2 - \sigma^2)\exp\{2\int_0^t l(s)\,ds\}. \tag{7}$$

Lemma 1. (of Gronwall-Bellman). If

$$|\omega(t)|^2 \leq \alpha^2 + 2\int_0^t l(s)|\omega(s)|^2\,ds,$$

then $|\omega(t)| \le \alpha e^{\int_0^t l(s)ds}$, where $\omega(t),\ t \ge 0$ is a measurable function and α is a non-negative number and $l(s)$ is a non-negative function.

Now we will introduce a notation

$$L(t) = e^{\int_0^t l(s)ds}.$$

Property 1. If $\rho \ge \sigma$, then the function $\lambda_{GB}(t,v)$ is continuous, nonnegative and defined for all the control functions $v(\cdot)$ such that satisfies the GB-constraint (3).

Property 2. If $\rho \ge \sigma$, then the following inequality is true for the function $\lambda_{GB}(t,v)$:

$$L(t)(\rho - \sigma) \le \lambda_{GB}(t,v) \le L(t)(\rho + \sigma).$$

Theorem. If $\rho > \sigma$ and $l(s) \ge 0$ in the Gr-game, then the Π_{GB}-strategy of the player $\mathbf{P}$ is winning on interval $[0, T_{GB}]$, where T_{GB} is a positive solution of an equation

$$\int_0^t L(s)ds = \frac{|z_0|}{\rho - \sigma}.$$

Proof. Suppose the pursuer choose the strategy (6) when the evader chooses any control function $v(\cdot) \in \mathbf{V}_{GB}$. Then according to the equations (1)-(2), we have the following Cauchy problem:

$$\dot{z} = -\lambda_{GB}(t, v(t))\xi_0, \quad z(0) = z_0.$$

Thus the following solution is found by the given initial conditions

$$z(t) = z_0 - \int_0^t \lambda_{GB}(s, v(s))\xi_0 ds$$

or

$$|z(t)|=|z_0|\left(1-\frac{1}{|z_0|}\int_0^t\left[(v(s),\xi_0)+\sqrt{(v(s),\xi_0)^2+\rho^2 e^{2\int_0^t l(s)ds}-|v(s)|^2}\right]ds\right).$$

According to the properties (1)-(2), we will form the following inequality

$$|z(t)|\le 1-\frac{1}{|z_0|}\int_0^t(\rho-\sigma)L(s)ds=\Lambda(t).$$

By solving an equation $\Lambda(t)=0$ we get a positive solution $t=T_{GB}$

$$1=\frac{1}{|z_0|}\int_0^t(\rho-\sigma)L(s)ds.$$

From this, we have

$$\frac{|z_0|}{(\rho-\sigma)}=\int_0^t L(s)ds.$$

This finishes the proof of the theorem.

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